Complete determination of circulator eigenvalues without transmission phase measurement

Ch. Schieblich and U. Goebel

Technische Universität Hamburg-Harburg, Arbeitsbereich Hochfrequenztechnik, Postfach 90 14 03, D-2100 Hamburg 90, West-Germany.

SUMMARY

A method has been developed to determine all scattering parameters or their eigenvalues of a symmetrical three-port junction. Vector measurements are reduced to reflection coefficient measurements taken at one port. A versatile minicomputer controlled system is presented for analyzing and optimizing symmetrical junction circulators of any kind.

INTRODUCTION

Usually circulator design concentrates on optimizing the magnitudes of the scattering parameters. Phase information is not specified because phase is difficult to measure. A complete knowledge of magnitudes and phases is, however, not only important for applications with critical delay times, but allows a better understanding of circulator operation and simplifies design. The scattering parameters can be converted to the scattering matrix eigenvalues. These are the reflection coefficients for in-phase, clockwise and counterclockwise rotating eigen-excitations. The eigenvalues have a concrete physical meaning, because they correspond to different eigenmodes of the ferrite resonator. Knowing these eigenvalues enables one to match the phase slopes of the 3 resonant circuits. Another application is to identify spurious resonances.

Direct vector measurements of the scattering parameters is a cumbersome task, because monitoring one reflection and two transmission coefficients requires extensive RF switching. Another problem is to find identical reference planes for transmission and reflection coefficients, a basic requirement for calculating consistent scattering matrix eigenvalues. Owen /1/ measured these eigenvalues by directly generating the eigen-excitation using power splitters and phase shifters. His measurement set-up requires perfect symmetry and has to be balanced very carefully in order to achieve constancy in amplitude and phase offsets (120°, 240°) at the three ports over a larger bandwidth.

NEW MEASUREMENT TECHNIQUE

Our approach avoids these problems at RF because phase measurements must only be done for one reflection coefficient with well-known terminations at the other ports. The scattering parameters or their eigenvalues are then evaluated by the computer which controls the network analyzer. The scattering matrix of a three-port with rotation symmetry is

$$S = \begin{bmatrix} s_{11} & s_{12} & s_{13} \\ s_{13} & s_{11} & s_{12} \\ s_{12} & s_{13} & s_{11} \end{bmatrix} \qquad (1)$$

If ports 2 and 3 are terminated by Γ_2 and Γ_3, the input reflection coefficient at port 1 reads

$$\Gamma_{in} = s_{11} + \frac{(\Gamma_2 + \Gamma_3)\,A + \Gamma_2\,\Gamma_3\,(B - 2s_{11}\,A)}{1 - (\Gamma_2 + \Gamma_3)\,s_{11} + \Gamma_2\,\Gamma_3\,(s_{11}^2 - A)}, \qquad (2)$$

$$A = s_{12}\,s_{13}\;,\quad B = s_{12}^3 + s_{13}^3.$$

With three sets of terminations Γ_2, Γ_3, the unknows s_{11}, A, and B can be calculated from the measured Γ_{in}. Matched loads and shorts may be used as terminations. The measurment for $\Gamma_2 = \Gamma_3 = 0$ (Fig. 1a) yields s_{11} directly. The other measurements are made with one port matched and the other shorted (Fig. 1b) and with both ports shorted (Fig. 1c). Interchanging Γ_2 and Γ_3 does not affect the input reflection, but may serve as a test for junction symmetry.

The set of linear equations (2) can be solved for s_{11}, A, B. With the above mentioned reflection coefficients this results in

$$s_{11} = \Gamma_{in}^{(o,o)}$$

$$A = -(\Gamma_{in}^{(-1,o)} - s_{11})\,(1 + s_{11})$$

$$B = (\Gamma_{in}^{(-1,-1)} - s_{11})\left[(1 + s_{11})^2 - A\right] + 2(1 + s_{11})\,A \qquad (3a)$$

Then s_{12} and s_{13} are given by

$$\left.\begin{matrix} s_{12}^3 \\ s_{13}^3 \end{matrix}\right\} = \frac{B}{2} \pm \sqrt{\left(\frac{B}{2}\right)^2 - A^3}, \quad s_{12} = A/s_{13}. \qquad (3b)$$

One cannot distinguish between s_{12} and s_{13} from one-port measurements alone. The missing information can, however, usually be taken from e.g. the

0149-645X/85/0000-0489 $01.00 © 1985 IEEE 1985 IEEE MTT-S Digest

direction of the circulation. The phase angles of s_{12} and s_{13} are unique except for multiples of 120°.

The S-matrix (1) can be converted to its eigenvalues

$$\begin{bmatrix} s_o \\ s_+ \\ s_- \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \end{bmatrix} \begin{bmatrix} s_{11} \\ s_{12} \\ s_{13} \end{bmatrix} , \alpha = e^{j120°} \quad (4)$$

without ambiguity. Their correspondence to the individual eigen-excitations remains, however, yet undetermined. Physcially, reflection measurements at one port lead to an equal excitation of the 3 eigenvectors. Hence a distinction of their individual contribution to the reflected wave is not possible. Fortunately this is not a severe restriction, because the in-phase eigenvalue can easily be identified in practical cases. Reciprocal junctions have degenerated rotating eigenmodes s_+ and s_-. In junction circulators, splitting between the phases of s_+ and s_- depends on the bias field, whereas s_o remains nearly unaffected. In the end, the succession of the eigenvalues is known from the sense of circulation.

ADDITIONAL AMPLITUDE MEASUREMENT

Evaluating (3) for highly nonreciprocal junctions such as circulators does not impose any problem; the same holds for reciprocal devices, because $s_{12} = s_{13}$ may be set a priori.

Junctions with a small nonreciprocity, such as detuned or weakly magnetized circulators, or circulators at out-of-band frequencies, show $s_{12} \approx s_{13}$, so that the difference in the square root of equ. (3) becomes small. This means that measurement errors are magnified with respect to the scattering matrix elements until an interpretation is no longer possible. A measurement technique for designing and optimizing circulators should, however, work for any junction.

These problems have been solved by an additional magnitude measurement of one transmission coefficient. This scalar value is read by the computer simultaneously with the input reflection coefficients by means of a detector at one of the matched loads. Port 3 being the monitored port, the transmission parameter is

$$g_3 = \frac{s_{13} + \Gamma_2 (s_{12}^2 - s_{11} s_{13})}{1 + \Gamma_2 \Gamma_3 (s_{11}^2 - s_{12} s_{13}) - (\Gamma_2 + \Gamma_3) s_{11}} \quad (5)$$

Measuring $|g_3|$ for the cases $\Gamma_2 = \Gamma_3 = 0$ and $\Gamma_2 = -1$, $\Gamma_3 = 0$ gives two additional equations for determining s_{13}.

$|g_3^{(o,o)}|$ for $\Gamma_2 = \Gamma_3 = 0$ yields $|s_{13}| = |g_3^{(o,o)}|$, and $|g_3^{(-1,o)}|$ for $\Gamma_2 = -1$, $\Gamma_3 = 0$ leads to

$$\cos(3 \cdot \varphi_{13} - \varkappa) = \frac{|s_{13}|}{2|K|} \left(|g_3^{(-1,o)}|^2 - |s_{13}|^2 - \frac{|K|^2}{|s_{13}|^4} \right) \quad (6)$$

with $s_{13} = |s_{13}| e^{j\varphi_{13}}$, $K = |K| e^{j\varkappa} = - \frac{A^2}{1+s_{11}}$

The terms in equ. (6) are not very sensitive to measuring errors, in particular for the case of weakly nonreciprocal junctions ($s_{12} \approx s_{13}$) while just the opposite holds for circulator junctions.

Hence is is convenient to switch between the methods of equ. (3) and equ. (6) depending on the magnitudes of s_{12}, s_{13}. Taking 0.1 as a limiting value gives good results. It must be noted, that equ. (6) has two solutions due to the ambiguity of the arccos-function, apart from the above-mentioned three-fold degeneracy. The correct root can be chosen by a comparison with equ. (3), or by using the frequency dependence, because the S-parameters and their eigenvalues must be continuous functions over frequency.

MEASUREMENT SYSTEM

Fig. 2 shows a complete set-up for circulator development. It consists of standard waveguide and / or coaxial components. Reflection is measured by a vector network analyzer (HP 8410 C), and transmission to both ports is simultaneously registered by a scalar analyzer (PM 1038). With this equipment, the magnitudes of the scattering parameters can be measured online. For phase measurements, a minicomputer (HP 9816) controls the instruments. The reflection coefficient measurements are corrected by an 8-term error model. At least three cycles are necessary (load/load; load/short; short/short). The short circuits in the reference planes are realized by movable shorting plates across the waveguide; in coaxial technique, SPDT-switches can be used. For improved accuracy, the waveguides can be shorted in a second plane about a quarter-wavelength apart. These measurements give redundant information, which is especially helpful near some singular points. The data can then be processed and displayed in various ways. e.g. switching between S -parameters and eigenvalues or shifting the reference plane is possible.

RESULTS

Two examples shall demonstrate the potentials of the method. First we examine a simple waveguide H-plane junction circulator with quarter-wave step transformers. Fig. 3a and c show the amplitudes of the S-parameters in the unmagnetized and the magnetized state, respectively. Fig. 3b shows the eigenvalues of the unmagnetized junction referred to the triangular boundary of the inner circulator. The eigenvalues $s_o = 1$, $s_+ = s_- = -1$ of the ideal parallel junction are modified by the transformer and the ferrite disc. The extension of the junction and the quarter-wave transformer lead to a dispersive s_o. The degenerate (rotating) eigenvalues s_+, s_- approach the s_o-curve at the band edges, and run through the low-Q E_{11}-resonance of the ferrite disc between them. At 12.4 GHz, a high-Q resonance is seen. It is excited by the rotating eigenvalues s_+, s_-, and corresponds to a higher-order ferrite mode. In Fig. 3d, the junction is biased for circulation. S_o remains unaffected, the center frequency of the s_- resonance decreases, that of s_+ increases, so that the 120° phase shift required for circulation is achieved. The slope of the s_+, s_--eigenvalues is matched to that of s_o, so that broadband behaviour is achieved. The spurious resonance above the operation band is split. The frequency corresponding to the s_- eigenvalue has moved down, in the same direction as the fundamental mode. This means that it corresponds to a ferrite mode with the same azimuthal index, probably with another height dependence.

The other example is a ferrite-loaded E-plane waveguide junction. Without bias field, the junction shows stopbands at 10.3 and 12.2 GHz (Fig. 4a). The eigenvalue diagram referred to the junction axis (Fig. 4b), allows an interpretation. The ideal E-plane junction has the eigenvalues $s_o = -1$, $s_+ = s_- = 1$. They are perturbed by two resonances. At 10.3 GHz, the rotating eigenvalues run through a series resonance causing $s_o = s_+ = s_- = -1$, whereas the upper stopband is caused by a parallel resonance of the in-phase system ($s_o = s_+ = s_- = 1$). Hence the lower resonance can be used for a reverse-type circulation /2/ if s_+ and s_- are split by 120°. This is shown in Fig. 4c, d. The amplitude characteristic corresponds exactly with that predicted in /2/. The resonance at 12.2 GHz is not affected by the bias field, because it is caused by an angular independent mode.

ACKNOWLEDGEMENT

The authors wish to thank the Deutsche Forschungsgemeinschaft for financial support.

REFERENCES

/1/ Owen: "The Identification of Modal Resonances in Ferrite Loaded Waveguide Y-Junctions and Their Adjustment for Circulation", Bell System Tech. J., vol. 51 (1972), pp. 595-627.

/2/ Goebel, Schieblich: "A Unified Equivalent Circuit Representation for H- and E-Plane Circulators", Proc. 13th EuMC 1983, pp. 249-251.

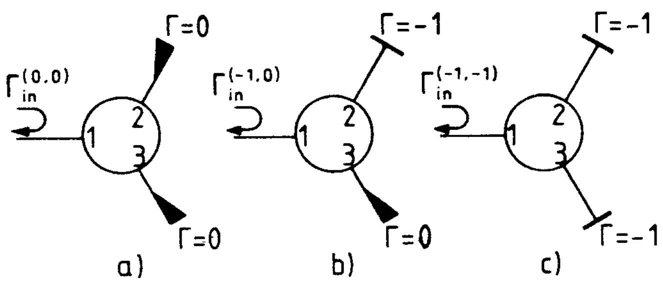


Fig. 1: Circuits for the three reflection coefficient measurements

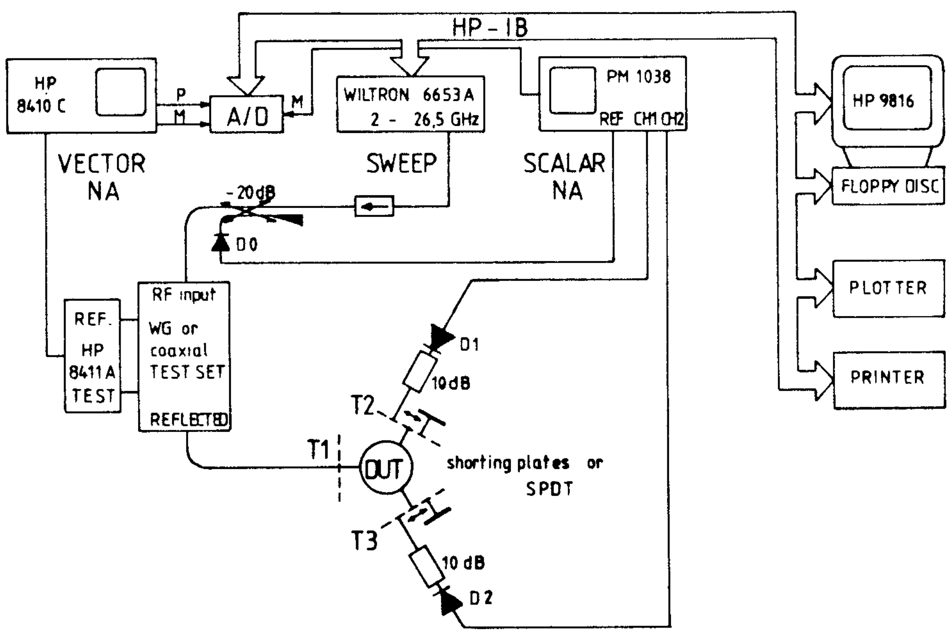
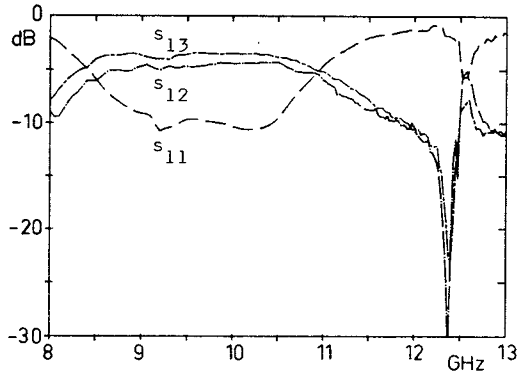
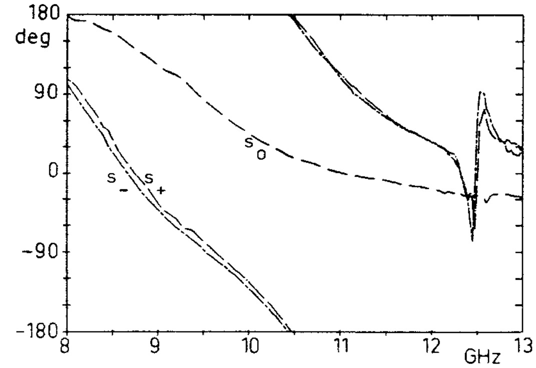


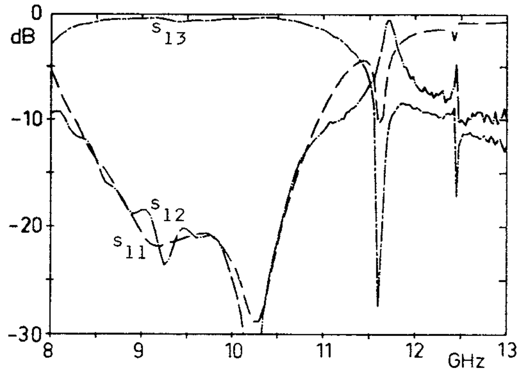
Fig. 2: The complete analyzer system



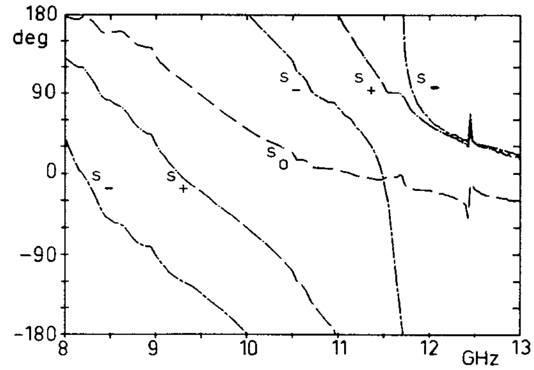
a) Scattering parameters without magnetisation



b) Phase of eigenvalues without magnetisation

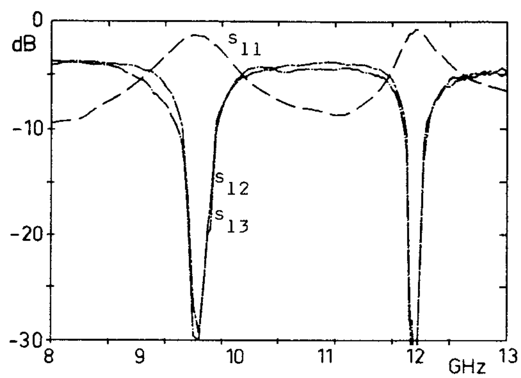


c) Scattering parameters with magnetisation

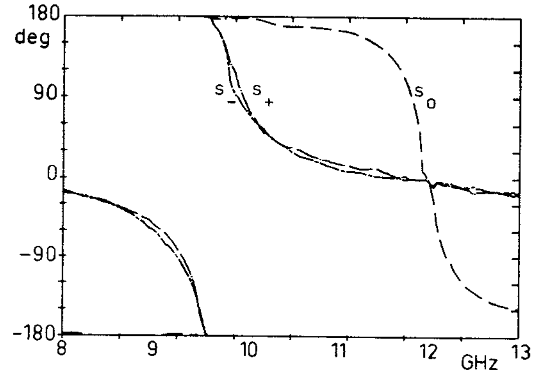


d) Phase of eigenvalues with magnetisation

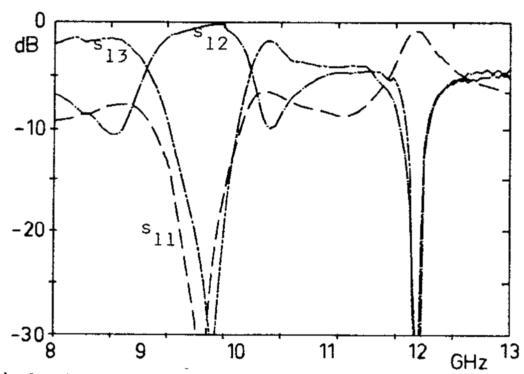
Fig. 3: Waveguide forward type circulator (H-plane)



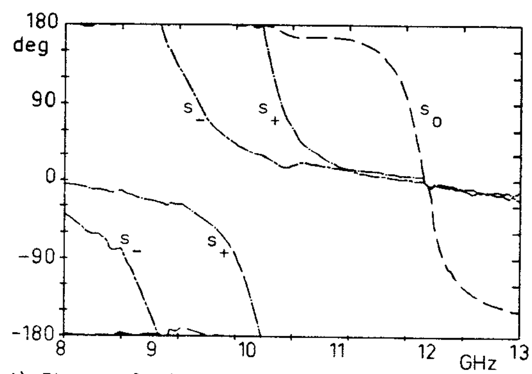
a) Scattering parameters without magnetisation



b) Phase of eigenvalues without magnetisation



c) Scattering parameters with magnetisation



d) Phase of eigenvalues with magnetisation

Fig. 4: Waveguide reverse type circulator (E-plane)